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Study of Double Diffusive MHD Natural Convective Flow from a Vertical Flat Plate in Porous Medium using Laplace Transform

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Abstract—*The present paper reports the analytical study of Double Diffusive MHD free convective flow in vertical flat plate in porous medium. The physical model is written in the form of coupled PDE. The similarity transformation technique is adopted to transform the PDE in to the system of ODE. The Laplace transform method is adopted to find the solution of system of ODE. The graphical approach is taken into account to explain the impact of distinguished physical parameter.*

Keywords: *MHD, Double Diffusive, Porous Medium, Laplace Transform.*

1. INTRODUCTION

Combined effect of heat and mass transfer by free convective flow in a fluid saturated porous medium has received remarkable attention during the last decades. This is only due to the importance of this process which leads to many engineering application like, geophysical and natural systems of practical interest such as geothermal energy utilization, thermal energy storage and recoverable systems and petroleum reservoirs. This has been shown effectively that, the simultaneous occurrence of heat and mass transfer between the fluxes, the driving potential is become more intricate nature. A clear understanding of the nature of the interaction between thermal and mass or solutal concentration buoyancy forces is necessary in order to control these processes. **Chaudhary and Jain [10]** studied the MHD flow past an infinite vertical oscillating plate through porous medium with the presence of free convection and mass transfer analytically by using Laplace-transform technique. **Mohamed [9]** analyzed the double-diffusive convection-radiation interaction for the unsteady MHD flow over a semi-infinite vertical moving porous plate embedded in a porous medium in the presence of thermal &solutal diffusion and heat generation. A numerical study of the unsteady free convection and mass transfer flow of an electrically conducting fluid past an infinite vertical porous plate in the presence of a transverse magnetic field was presented by **Shariful et. [6]**. **Bukhari, [2]**applied a linear stability analysis, with basic flowa fluid layer overlying a porous layer. **Saha&Hossain [11]** studied numerically the laminar doubly diffusive free convection flows along an isothermal vertical finite plate immersed in a stable thermally stratified fluid by using an implicit finite difference method and local non-similarity method. **Hajri et. al. [3]** presented a numerical simulation for the steady double-diffusive natural convection in a triangular cavity by using equal finite elements method. A numerical study was presented from **Mamouet. al. [5]** for the unsteady double-diffusive convection in a two-dimensional horizontal confined enclosure by using the finite element technique.. **Khanafer and Vafai, [4]** presented a numerical study of mixed-convection heat and mass transport in a lid-driven square enclosure filled with a non-Darcian fluid-saturated porous medium by using the finite volumes technique.

2. THE MATHEMATICAL MODEL

The model of the problem is presented Fig1. Here, this has been considered a steadystate, two dimensional laminar natural convective boundary layer flow of a incompressible vertical flat plate. The plate is electrically and fluid is viscosity dependent. This also be assumed that the temperature past a semi infinite vertical impermeable flat plate in presence of uniformly distributed transverse magnetic filed of strength H_o in the porous medium. The impact is take nunder the Bousineqess approximation. The physical conditions are taken in X-Y plane. The boundary layer flow is taken care in Y-direction only. Keeping in view of the above conditions .The basic equations are given below with respect to x and y axis.

3. THE GOVERNING EQUATION

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma u H_0^2}{\rho_\infty}-\frac{\mu}{k_p}u (2)$$

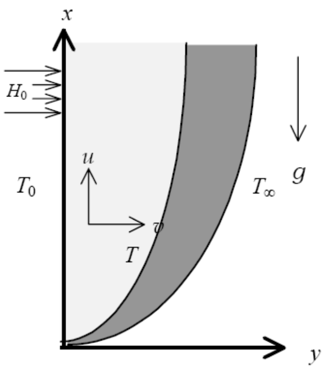
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho_\infty C_p}\frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}=\varepsilon\frac{\partial^2 S}{\partial y^2} \quad (4)$$

with the boundary conditions

$$u = v = 0, T = T_0, S = S_0 \ at\ y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty \ at\ y \ \rightarrow \infty \quad (5)$$



The of physical model of problem

Here *u, v* is the velocity components associated with the direction of increase of coordinates *x* and *y* measured along and normal to the vertical plate. *T* is the temperature of the fluid in the boundary layer, *g* is the acceleration due to gravity, β is the coefficient of thermal expansion, κ is the thermal conductivity, $\rho_\infty$is the density of the fluid, *cp* is the specific heat at constant pressure and $T_\infty$is the temperature of the ambient fluid and ν the kinematics viscosity of the fluid. From the continuity equation (1) we consider the velocity normal t the plate is of the form $v = -V0$ .Now we introduce the following transformations to the equation (2) and (3)

4. SIMILARITY TRANSFORM PARAMETER

$$Y = \frac{yV_0}{v},\ U = \frac{u}{U_0},\ \theta = \frac{(T - T_\infty)}{(T_0 - T_\infty)},\ \theta = \frac{(S - S_\infty)}{(S - S_\infty)},$$

$$Gr_T, = \frac{g\beta_T(T_0 - T_\infty)v}{U_0 V_0^2},\ P_r = \frac{\mu C_p}{\kappa},\ M = \frac{\sigma v H_0^2}{\rho_\infty V_0^2}$$

$$Gr_S, = \frac{g\beta_S(S_0 - S_\infty)v}{U_0 V_0^2} \quad (6)$$

5. THE TRANSFORMED GOVERNING EQUATION

$$U''(y) + U'(y) - \left(M + \frac{1}{Da}\right)U(y) + Gr_T\emptyset(y) + Gr_S\emptyset(y) = 0 \quad (7)$$

$$\theta''(y) + P_r\theta'(y) = 0 \quad (8)$$

$$\emptyset''(\boldsymbol{y}) + Sc\emptyset'(y) = 0 \quad (9)$$

with the boundary conditions

$$U = 0, \theta = \emptyset = 1 \ at\ Y = 0$$

$$\lim_{y\to\infty} U(y) = 0,\ \lim_{y\to\infty} \theta(y) = 0,\ \lim_{y\to\infty} \emptyset(\boldsymbol{y}) = \mathbf{0}$$

6. ANALYTICAL SOLUTION

The analytical solution of equations (7)-(9) along with the boundary conditions is performed using Laplace transform.

$$\phi''(y) + S_c\phi'(y) + \psi\phi(y) = 0, \phi(0) = 1, \lim_{y\to\infty}\phi(y) = 0$$

Let $\phi'(0) = c_{2, s^2 > 4\psi}$

$$L\{\phi'(y) + S_c\phi'(y) + \psi\phi(y)\} = 0$$

$$s^2\bar{\phi}(s) - s\phi(0) - \phi'(0) + S_c\left(s\bar{\phi}(0) - \phi(0)\right) + \psi s\bar{\phi}(s) = 0$$

$$(s^2 + sS_c + \psi)\bar{\phi}(s) = s + c_2 + S_c$$

$$\bar{\phi}(s) = \frac{(s + S_c) + c_2}{s^2 + s\frac{S_c}{2} + \frac{(S_c)^2}{4} + (\psi - \frac{(S_c)^2}{4})}$$

$$\bar{\phi}(s) = \frac{\left(s + \frac{S_c}{2}\right) + (c_2 + \frac{S_c}{2})}{(s + \frac{S_c}{2})^2 + (\frac{(\sqrt{S_c{}^2 - 4\psi})}{2})^2}$$

$$= \frac{\left(s + \frac{Sc}{2}\right)}{(s + \frac{Sc}{2})^2 - (\frac{(\sqrt{Sc^2 - 4\psi})}{2})^2} + \frac{(c_2 + Sc)}{\frac{(\sqrt{Sc^2 - 4\psi})}{2}} \frac{(\frac{\sqrt{Sc^2 - 4\psi}}{2})}{(s + \frac{Sc}{2})^2 - (\frac{\sqrt{Sc^2 - 4\psi}}{2})^2}$$

$$\bar{\phi}(s) = \frac{\left(s + \frac{Sc}{2}\right)}{(s + \frac{Sc}{2})^2 - (\frac{(\sqrt{Sc^2 - 4\psi})}{2})^2} + \frac{(c_2 + Sc)}{\frac{(\sqrt{Sc^2 - 4\psi})}{2}} \frac{(\frac{\sqrt{Sc^2 - 4\psi}}{2})}{(s + \frac{Sc}{2})^2 - (\frac{\sqrt{Sc^2 - 4\psi}}{2})^2}$$

$$L^{-1}\{\bar{\phi}(s)\} = L^{-1}\{\frac{\left(s + \frac{Sc}{2}\right)}{\left(s + \frac{Sc}{2}\right)^2 - (\frac{\left(\sqrt{Sc^2 - 4\psi}\right)}{2})^2} + \frac{(c_2 + Sc)}{\frac{\left(\sqrt{Sc^2 - 4\psi}\right)}{2}} \frac{\left(\frac{\sqrt{Sc^2 - 4\psi}}{2}\right)}{(s + \frac{Sc}{2})^2 - (\frac{\sqrt{Sc^2 - 4\psi}}{2})^2}\}$$

$$\phi(y) = \left(e^{-\frac{Scy}{2}}\right)\left(\cosh\left(\frac{\sqrt{Sc^2 - 4\psi}}{2}y\right)\right) + \frac{2c_2 + Sc}{\sqrt{Sc^2 - 4\psi}} e^{-\frac{Scy}{2}}(\sinh\left(\frac{\sqrt{Sc^2 - 4\psi}}{2}y\right))$$

$$= e^{-\frac{Scy}{2}}\left[\frac{e^{\frac{\sqrt{Sc^2-4\psi}}{2}}+e^{-\frac{\sqrt{Sc^2-4\psi}}{2}y}}{2}\right]$$

$$+\frac{2c_2+Sc}{\sqrt{Sc^2-4\psi}}(e^{-\frac{Scy}{2}})[e^{\frac{\sqrt{Sc^2-4\psi}}{2}y}$$

$$-e^{-\frac{\sqrt{Sc^2-4\psi}}{2}y}]$$

$$\phi(y)=\frac{e^{-\frac{Scy}{2}+\frac{\sqrt{Sc^2-4\psi y}}{2}}}{2}\left[\left(1+e^{-\sqrt{Sc^2-4\psi}y}\right)\right]$$

$$+\frac{2c_2+Sc}{\sqrt{Sc^2-4\psi}}\left(1-e^{-\sqrt{Sc^2-4\psi}y}\right)$$

$$\phi(y)=\frac{1}{2}e^{\frac{(-Sc+\sqrt{Sc^2-4\psi})}{2}y[1+e^{-\sqrt{Sc^2-4\psi}y}+\frac{2c_2}{\sqrt{Sc^2-4\psi}}\left(1-e^{-\sqrt{Sc^2-4\psi}y}\right)]}$$

$$\lim_{y\to\infty}\phi(y)$$

$$=\frac{1}{2}e^{\frac{(-Sc+\sqrt{Sc^2-4\psi})}{2}y[1+e^{-\sqrt{Sc^2-4\psi}y}+\frac{2c_2}{\sqrt{Sc^2-4\psi}}\left(1-e^{-\sqrt{Sc^2-4\psi}y}\right)]}$$

$$\frac{1}{2}1\left(1+0+\frac{2c_2}{\sqrt{Sc^2-4\psi}}(1-0)\right)=0$$

$$c_2=-\frac{\sqrt{Sc^2-4\psi}}{2}$$

$$Let\ \theta'(0)=k_1,\phi'(0)=k_2,U'(0)=k_3$$

$$L\{\theta''(y)+Pr\theta'(y)\}=L\{0\}$$

$$L\{\theta''(y)\}+PrL\{\theta'(y)\}=0$$

$$s^2\bar{\theta}(s)-s\theta(0)-\theta'(0)+Pr\left(s\,\bar{\theta}(s)-\theta(0)\right)=0$$

$$(s^2+Prs)\bar{\theta}(s)=s+Pr+k_1$$

$$\bar{\theta}(s)=\frac{s+Pr+k_1}{s(s+Pr)}$$

$$\bar{\theta}(s)=\left(1+\frac{k_1}{Pr}\right)\frac{1}{s}-\frac{k_1}{Pr}\left(\frac{1}{s+Pr}\right)$$

$$L^{-1}\{\bar{\theta}(s)\}=\left(1+\frac{k_1}{Pr}\right)L^{-1}\left\{\frac{1}{s}\right\}-\frac{k_1}{Pr}L^{-1}\left\{\frac{1}{s+Pr}\right\}$$

$$\theta(y)=\left(1+\frac{k_1}{Pr}\right)1-\frac{k_1}{Pr}e^{(-Pry)}$$

$$\theta(y)=1+\left(\frac{k_1}{Pr}\right)\left(1-e^{(-Pry)}\right)$$

$$But\ \lim_{y\to\infty}\theta(y)=\lim_{y\to\infty}\left\{1+\frac{k_1}{Pr}\left(1-e^{(-Pry)}\right)\right\}$$
$$0$$
$$-\infty$$

$$0=1+\frac{k_1}{Pr}(1-1)$$

$$k_1=-Pr,$$

$$\theta(y)=e^{-Pry}$$

7. RESULT AND DISCUSSION

In order to understand the physics behind the phenomena the graphical representation of distinguished physical parameter is given in this section. The main emphasis is given to the velocity variation of flow dynamics. The following section demonstrates the influence of different parameter in MHD double diffusive free convective fluid flow from a vertical flat plate in porous media with uniform viscosity and uniform thermal conductivity, in presence of uniform transverse magnetic field along an impermeable vertical flat plate. The similarity solution used for the demonstration the non-dimensional velocity.

Figure 1 shows the effect of Hartmen number (M) while fixing the other physical parameters. This has been observed from the figure that, as we increase the value of M. The velocity is decreased drastically. However, the flow is asymptotically stable for the large characteristics values.

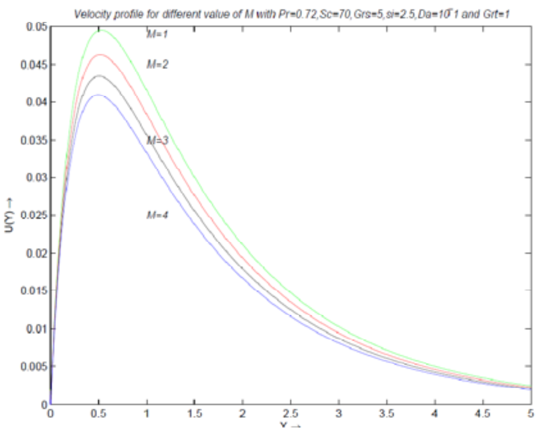


Figure 1: Effect if Hartmen Number (M)

Figure 2, shows the impact of solutalgrashof number on velocity. This has been observed from the figure that, the impact of high grashof is affected the velocity. But, this is also can be seen that that after certain point the profile is crosses each other. As a results this can be conclude that the very near to boundary the solutal effect is much effected, whereas far from the boundary the effect is not much significant ot negligible.

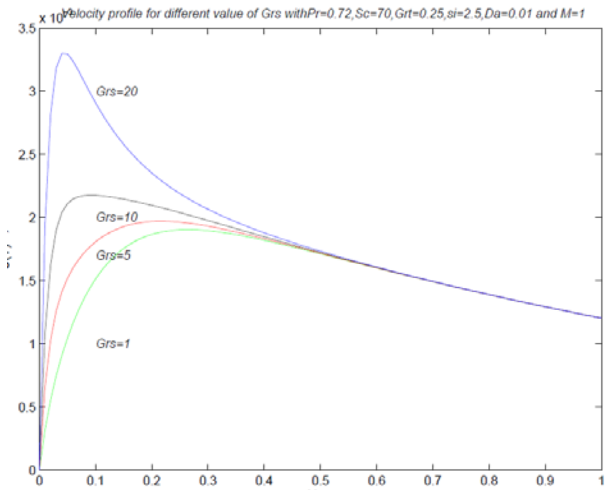


Figure 2 Effect of SolutalGrashof Number (Gr_S)

Figure 3, shows the effect of the Darcy number (Da) on velocity. here we see that the as we increase the darcy number the velocity is being decreases which is effective for the porous media here in the special case as Da=1, the velocity is very high which is shows that the as we entered in the viscous media, velocity is higher than the prous media, now the darcy number is

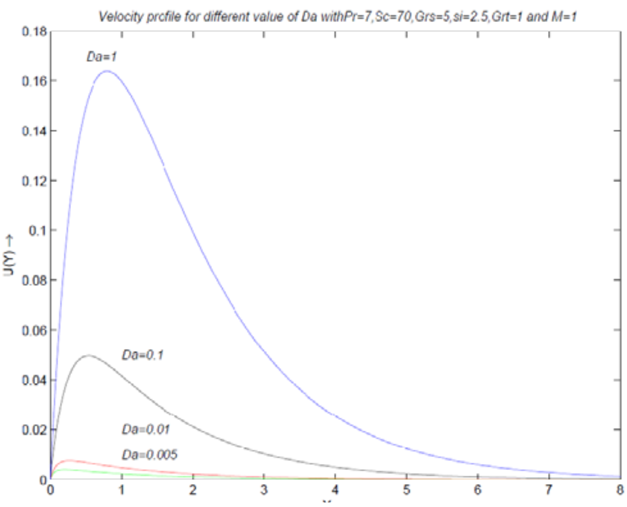


Figure 3 Effectof Darcy Number (Da)

goingdecreases as Da=0.1, 0.01, 0.001, we saw that the velocity is vanish in the case of Da=0.001, i.e there is velocity for the air flow,

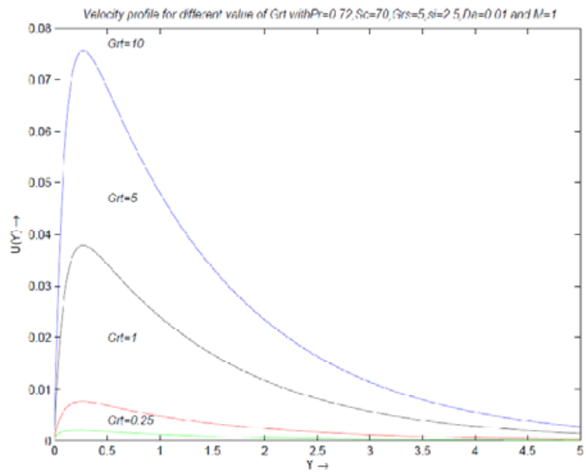


Figure 4: Effect of Thermal Grashof Number (Gr_T)

In last the figure the effect of thermal grashof number is depicted. This has been observed from the figure that, for the higher values of thermal garshof the velocity is high. However, the velocity is increased for the enhancing the value of thermal grahof number

8. CONCLUSION

In the presented manuscript the Laplace transform method is successfully implemented. The results are cross verified with the published problem in special case. This can be conclude from the figures that the Hartman number is act as retardation force, whereas the grashof number is act as booster of flow mechanism.

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